K – Multiset Variance

Memory limit: 1024 MB
Time limit: 2 s

EUC 2026
8.02.2026

We have n multisets of integers. We create a nonempty multiset by taking each multiset from the input in whole arbitrarily many times (possibly 0 times).

For example, from input multisets $\{1, 3\}, \{1, 2, 2, 3\}, \{3, 5\}$, we can create a multiset $M = \{1, 3, 1, 3, 1, 2, 2, 3\}$ by taking the first one twice, the second one once, and not taking the third one.

What is the minimum and maximum variance of a multiset that we can achieve?

Recall that the mean $\mathrm{E}(X)$ and variance $\mathrm{Var}(X)$ of a multiset $X = \{x_1, x_2, \ldots, x_k\}$ are defined as follows:

$$\mathrm{E}(X) = \frac{1}{k}\sum_{i=1}^{k} x_i, \qquad \mathrm{Var}(X) = \frac{1}{k}\sum_{i=1}^{k} (x_i - \mathrm{E}(X))^2.$$

For instance, the mean and variance of the above multiset M are:

$$\mathrm{E}(M) = \tfrac{1}{8}(1+3+1+3+1+2+2+3) = \tfrac{16}{8} = 2,$$
$$\mathrm{Var}(M) = \tfrac{1}{8}((-1)^2+1^2+(-1)^2+1^2+(-1)^2+0^2+0^2+1^2) = \tfrac{6}{8} = \tfrac{3}{4}.$$

Input

The first line of the input contains one integer t ($1 \le t \le 1000$), denoting the number of test cases.

Each test case starts with a line containing one integer n ($1 \le n \le 10^5$), denoting the number of multisets.

The next n lines describe the multisets. Each line starts with an integer k ($1 \le k \le 10$) denoting the size of the multiset, followed by a sequence of k integers from the range $[-2 \cdot 10^5, 2 \cdot 10^5]$ representing the elements of the multiset.

The sum of the sizes of all multisets over all test cases does not exceed 10^6.

Output

Output exactly t lines containing answers to subsequent test cases.

The answer consists of two real numbers separated by a space, the minimum and maximum variance. The allowable relative or absolute error is 10^{-11}. That is, if you output S and the correct exact result is R, then it must hold that $|S - R| \le 10^{-11} \cdot \max(1, R)$. You may print at most 20 digits after the decimal point.

It can be shown that there is always a solution that achieves the minimum variance and a solution that achieves the maximum variance.

ICPC Global Sponsor Programming Tools: JETBRAINS
ICPC Diamond Multi-Regional Sponsor: HUAWEI
ICPC Titanium Multi-Regional Sponsor: Jane Street
ICPC Platinum Sponsors: Jump Trading, Revolut
ICPC Gold Sponsors: Citadel | Citadel Securities, HRT, Google
ICPC Sponsor: pinely

K – Multiset Variance

Memory limit: 1024 MB
Time limit: 2 s

Example

For the input data:

```
3
3
2 1 3
4 1 2 2 3
2 3 5
2
3 -3 0 0
3 -2 -2 1
2
2 1 3
1 5
```

a correct result is:

```
0.5 2
2 2
0 2.777777777778
```

Explanation: In the first test case, the minimum variance (equal to $\frac{1}{2}$) is achieved for the multiset $\{1, 2, 2, 3\}$, and the maximum (equal to 2) for the multiset $\{1, 3, 3, 5\}$.

In the second test case, both minimum and maximum variance (equal to 2) is obtained for the multiset $\{-3, 0, 0, -2, -2, 1\}$.

In the third test case, the minimum variance (equal to 0) is achieved for the multiset $\{5\}$, and the maximum (equal to $\frac{25}{9}$) for the multiset $\{1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 5, 5, 5, 5, 5, 5, 5, 5\}$.

ICPC Global Sponsor Programming Tools: JETBRAINS
ICPC Diamond Multi-Regional Sponsor: HUAWEI
ICPC Titanium Multi-Regional Sponsor: Jane Street
ICPC Platinum Sponsors: Jump Trading, Revolut
ICPC Gold Sponsors: CITADEL | CITADEL Securities, HRT hudson river trading, Google
ICPC Sponsor: pinely