Solutions

ACM/ICPC 2015 EC-Final

Practice Round

A.Dynamic Grid

Google APAC 2016 University Graduates Test Round D Problem A

Change the color and BFS.

B. IP Summarization

Google APAC 2016 University Graduates Test Round D Problem C

Binary tree.

Create the nodes, if both 2 children are existed, merge them to the parent.

C. Virtual Rabbit

Google APAC 2016 University Graduates Test Round D Problem D

Greedy on feeding time. Get the circle section.

Corner cases handling:

- Maybe don’t need to feed on the last day.
- Even if sleeping time exceeds feeding time, it still could survive 1 day.

EC-Final

Accepted: 274

A. Boxes and Balls

- The answer must be `m(m+1)/2` for some integer m. Find the largest m satisfying `m(m+1)/2 ≤ N`.
- Solution 1: solve the equation. Note that `double` doesn't have enough precision for 10^{18}.
- Solution 2: binary search.

Accepted: 78

B. Business Cycle

Binary search

Sum <=0: direct simulation

Sum >0: simulate the first 2 circles and add sum. Judge the value on last circle.

Accepted: 2

C. Suffixes and Palindromes

Combine of Suffixes sort and Manacher's algorithm

=> A[i] >= A[j] or A[i] > A[j]

difference constraints or greedy algorithm

Accepted: 151

D. Change

- You can use the vending machine multiple times.
- For any pair (A, B), you only need to spend 0.01 CNY once or twice.
- Solution 1: use search or DP to check if it is possible to get (A-1) without using B (or anything that combine to B).
- Solution 2:

  if $B \in \{0.01, 0.1, 1, 10\}$ and $A >= 5B$: 0.02;
  otherwise, 0.01.

Accepted: 2

E. Colorful Floor

Cyclic group C_n act on set X, color set K.

$$|K^X/C_n| = \frac{1}{n}\sum_{i|n}\varphi(i)|K|^{n/i}$$

Group direct product $C_n \times C_m$ act on set $X \times Y$, color set K.

$$|K^{X\times Y}/C_n \times C_m| = \frac{1}{nm}\sum_{\substack{i|n\\ j|m}}\varphi(i)\varphi(j)K^{\frac{nm}{\mathrm{lcm}(i,j)}}$$

E. Colorful Floor

Consider equivalent classes(orbits) under group action G.When an orbit $\bar{x} \in K^X/G$ is fixed under the given color permutation p, we have

$$p(\bar{x}) = \bar{x}$$

Recall that **Burnside's lemma** gives

$$|K^X/G| = \frac{1}{|G|} \sum_{g \in G} \#\{x \in K^X \mid xg = x\}$$

In our case, the modified Burnside's lemma gives

$$\#\{\bar{x} \in K^X/G \mid p(\bar{x}) = \bar{x}\} = \frac{1}{|G|} \sum \#\{x \in K^X \mid xg = px\}$$

Accepted: 2

E. Colorful Floor

Suppose $g = (g_1\ g_2\ \dots\ g_k)(\dots)\dots(\dots)$ and $x = (c_1\ c_2\ \dots\ c_k)(\dots)\dots(\dots)$, condition

$$xg = px$$

gives

$$p(c_1) = c_2,\ p(c_2) = c_3\ \cdots\ , p(c_k) = c_1$$

Hence for any color c in the cycle $p^k(c) = c$.

Back to our problem

$$\text{ans} = \frac{1}{nm} \sum_{\substack{i|n \\ j|m}} \varphi(i)\varphi(j)\ \mathrm{K}(i,j)^{\frac{nm}{\mathrm{lcm}(i,j)}}$$

Accepted: 83

F. Hungry Game of Ants

<- | -> | -> | -> | <- | -> | <- | <-

$$dp[i] = \sum_{\substack{j \\ j(j+1)>=i(i+1)/2}} dp[j] \qquad \text{when } i > k$$

$$dp[i] = \sum_{\substack{j \\ j(j+1)<i(i+1)/2}} dp[j] \qquad \text{when } i = k$$

$$dp[i] = 2^i \qquad \text{when } i < k$$

$$ans = dp[n]$$

Accepted: 0

G. Legacy of Void

Order Statistic

If we have the distribution of Sum W_i for each simulation. Then we can treat it as a binomial distribution.

Let

P1 = Pr(x < X).

$$P(X_{(k)} = x) = P(X_{(k)} \leq x) - P(X_{(k)} < x),$$
$$= \sum_{j=0}^{n-k} \binom{n}{j} \left(p_3^j (p_1 + p_2)^{n-j} - (p_2 + p_3)^j (p_1)^{n-j}\right),$$

Then

Accepted: 0

G. Legacy of Void

- To get the distribution for each Sum W_i
- Naive solution is tree dp.
- Complexity O(m * (n + m))
- Got TLE.
- FFT ??? -> May be

Given root, get the distributions for blocks include this root.

We can got the DFS order of this tree.

DP from the root by this order.

If we don’t select current node, then we will jump all the nodes in this current sub-tree.

In this order there is only one size of each node which is W_i.

Then we have a O(nm) approach for each root.

Accepted: 1

H. Open Face Chinese Poker

DP[i][j][k]

I: used card bit of 14 cards

J: The score and rank of 3 cards.

- Non-zero score: score +1
- zero score pair: 1
- High card: 0

K: The rank of lower 5 cards.(7 different ranks+1 no score rank).

O(2^14 * 24 * 8)

Pre-Cal C(14,5) cards rank.

Accepted: 1

I.Champions League

DP one country by one country

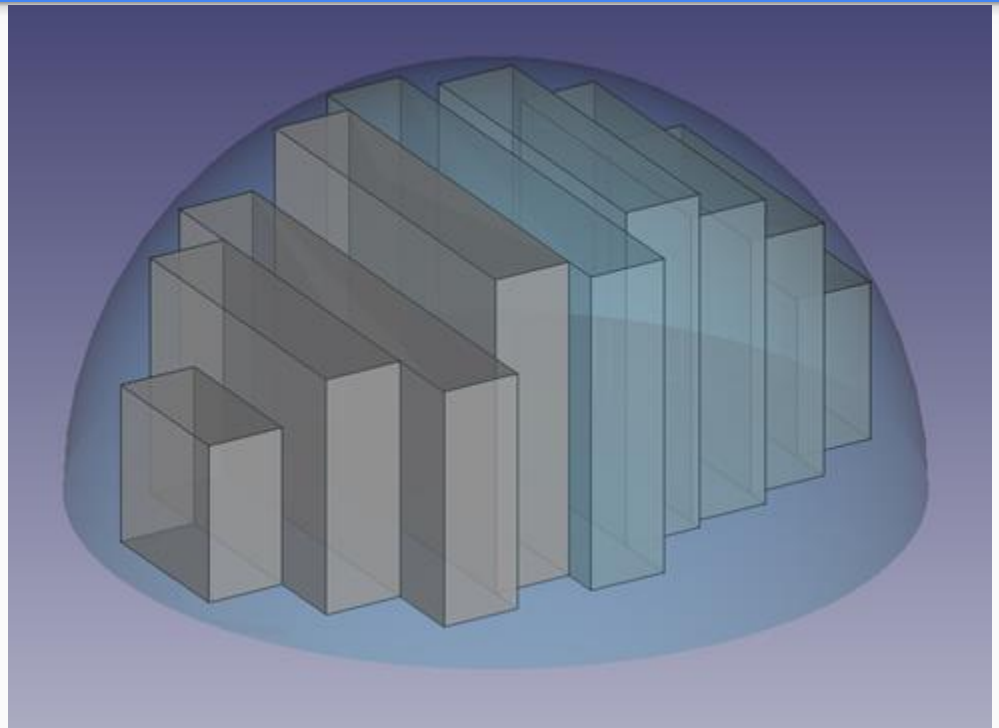
Naive State: 2^32

Keypoint: when DP at a specific country, #team in each level is unique.

Actual State: C(8,4)^4

Accepted: 41

J. Dome and Steles



Accepted: 41

J. Dome and Steles

- For stele *i*, the minimum height at its position is:

$$r_i = \mathrm{sqrt}(\max(a_i,b_i)^2/4+\min(a_i,b_i)^2).$$

- Use binary search to get the minimum radius.

- For a given radius R, calculate the range you can place a stele $[p_i, -p_i]$ from the center, where $p_i = \mathrm{sqrt}(R^2 - r_i^2)$. Then place the steles according to p_i from the largest to the smallest. If you can place all steles, R is valid.

Accepted: 0

K. Convex Polyhedron

Projection of a Convex Polyhedron

First, get the 3D convex hull of the point set.

There are O(N^3) possible normal vectors. For each candidate vector, find all the surfaces of the convex hull that can be “lit” by the vector. Sum these surfaces’ normal vector with surface area as the weight. The sum vector is the local optimal normal vector.

Accepted: 222

L. Multiplication Table

No number: Yes

1 number: get all factors, and judge the X and Y are enough.

2 or more numbers:

1. Solve equations, unique answer. Check it.
2. Get one’s factors and check on the other, unique answer. Check it.

Accepted: 268

M. November 11th

-------X-----X--X

Ans1 = (len+1) / 2

Ans2 = (len+2) / 3